

Uitwerkingen hoofdstuk 6

Wiskunde 1

6.1

- a. $\frac{a+3}{a-3} \rightarrow \text{term 1: } a, \text{ term 2: } 3 \rightarrow \frac{a}{a-3} + \frac{3}{a-3}$
- b. $\frac{2a+3b}{a-b} \rightarrow \text{term 1: } 2a, \text{ term 2: } 3b \rightarrow \frac{2a}{a-b} + \frac{3b}{a-b}$
- c. $\frac{a^2+3a+1}{a^2-3} \rightarrow \text{term 1: } a^2, \text{ term 2: } 3a, \text{ term 3: } 1 \rightarrow \frac{a^2}{a^2-3} + \frac{3a}{a^2-3} + \frac{1}{a^2-3}$
- d. $\frac{2a-b+3}{ab-3} \rightarrow \text{term 1: } 2a, \text{ term 2: } -b, \text{ term 3: } 3 \rightarrow \frac{2a}{ab-3} - \frac{b}{ab-3} + \frac{3}{ab-3}$
- e. $\frac{2-5a}{b-a^3} \rightarrow \text{term 1: } 2, \text{ term 2: } -5a \rightarrow \frac{2}{b-a^3} - \frac{5a}{b-a^3}$

6.2

- a. $\frac{a^2+b^2}{a^2-b^2} \rightarrow \text{term 1: } a^2, \text{ term 2: } b^2 \rightarrow \frac{a^2}{a^2-b^2} + \frac{b^2}{a^2-b^2}$
- b. $\frac{ab+bc-ca}{a-2b} \rightarrow \text{term 1: } ab, \text{ term 2: } bc, \text{ term 3: } -ca \rightarrow \frac{ab}{a-2b} + \frac{bc}{a-2b} - \frac{ca}{a-2b}$
- c. $\frac{b^2-1}{a^2-1} \rightarrow \text{term 1: } b^2, \text{ term 2: } -1 \rightarrow \frac{b^2}{a^2-1} - \frac{1}{a^2-1}$
- d. $\frac{4abc+5}{c-ab} \rightarrow \text{term 1: } 4abc, \text{ term 2: } 5 \rightarrow \frac{4abc}{c-ab} + \frac{5}{c-ab}$
- e. $\frac{(5ab^2-abc)}{(ab-c)} \rightarrow \text{term 1: } 5ab^2, \text{ term 2: } -abc \rightarrow \frac{5ab^2}{ab-c} - \frac{abc}{ab-c}$

6.3

- a. $\frac{1}{a-3} - \frac{1}{a+3} = \frac{1*(a+3)}{(a-3)*(a+3)} - \frac{1*(a-3)}{(a+3)*(a-3)} = \frac{a+3}{a*a+a*3-3*a-3*3} - \frac{a-3}{a*a+a*(-3)+3*a+3*(-3)} =$
 $\frac{a+3}{a^2+3a-3a-9} - \frac{a-3}{a^2-3a+3a-9} = \frac{a+3}{a^2-9} - \frac{a-3}{a^2-9} = \frac{((a+3)-(a-3))}{a^2-9} = \frac{a+3-a+3}{a^2-9} = \frac{6}{a^2-9}$
- b. $\frac{1}{a-3} + \frac{1}{a+3} = \frac{1*(a+3)}{(a-3)*(a+3)} + \frac{1*(a-3)}{(a+3)*(a-3)} = \frac{a+3}{a*a+a*3-3*a-3*3} + \frac{a-3}{a*a+a*(-3)+3*a+3*(-3)} =$
 $\frac{a+3}{a^2+3a-3a-9} + \frac{a-3}{a^2-3a+3a-9} = \frac{a+3}{a^2-9} + \frac{a-3}{a^2-9} = \frac{((a+3)+(a-3))}{a^2-9} = \frac{a+3+a-3}{a^2-9} = \frac{2a}{a^2-9}$
- c. $\frac{2}{a-3} - \frac{1}{a+3} = \frac{2*(a+3)}{(a-3)*(a+3)} - \frac{1*(a-3)}{(a+3)*(a-3)} = \frac{2a+6}{a*a+a*3-3*a-3*3} - \frac{a-3}{a*a+a*(-3)+3*a+3*(-3)} =$
 $\frac{2a+6}{a^2+3a-3a-9} - \frac{a-3}{a^2-3a+3a-9} = \frac{2a+6}{a^2-9} - \frac{a-3}{a^2-9} = \frac{((2a+6)-(a-3))}{a^2-9} = \frac{2a+6-a+3}{a^2-9} = \frac{a+9}{a^2-9}$

$$\begin{aligned}
 \text{d. } \frac{1}{a-3} + \frac{a}{a+3} &= \frac{1*(a+3)}{(a-3)*(a+3)} + \frac{a*(a-3)}{(a+3)*(a-3)} = \frac{a+3}{a*a+a*3-3*a-3*3} + \frac{a^2-3a}{a*a+a*(-3)+3*a+3*(-3)} = \\
 &= \frac{a+3}{a^2+3a-3a-9} + \frac{a^2-3a}{a^2-3a+3a-9} = \frac{a+3}{a^2-9} + \frac{a^2-3a}{a^2-9} = \frac{((a+3)+(a^2-3a))}{a^2-9} = \frac{a+3+a^2-3a}{a^2-9} = \frac{a^2-2a+3}{a^2-9}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } \frac{a}{a-3} - \frac{a}{a+3} &= \frac{a*(a+3)}{(a-3)*(a+3)} - \frac{a*(a-3)}{(a+3)*(a-3)} = \frac{a^2+3a}{a*a+a*3-3*a-3*3} - \frac{a^2-3a}{a*a+a*(-3)+3*a+3*(-3)} = \\
 &= \frac{a^2+3a}{a^2+3a-3a-9} - \frac{a^2-3a}{a^2-3a+3a-9} = \frac{a^2+3a}{a^2-9} - \frac{a^2-3a}{a^2-9} = \frac{((a^2+3a)-(a^2-3a))}{a^2-9} = \frac{a^2+3a-a^2+3a}{a^2-9} = \frac{6a}{a^2-9}
 \end{aligned}$$

6.4

$$\begin{aligned}
 \text{a. } \frac{(a+1)*(a+3)}{(a-2)*(a+3)} - \frac{(a-1)*(a-2)}{(a+3)*(a-2)} &= \frac{a*a+a*3+1*a+1*3}{a*a+a*3-2*a-2*3} - \frac{a*a+a*(-2)-1*a-1*(-2)}{a*a+a*(-2)+3*a+3*(-2)} = \frac{a^2+4a+3}{a^2+a-6} - \frac{a^2-3a+2}{a^2+a-6} = \\
 &= \frac{(a^2+4a+3)-(a^2-3a+2)}{a^2+a-6} = \frac{a^2+4a+3-a^2+3a-2}{a^2+a-6} = \frac{7a+1}{a^2+a-6}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{(a+1)*(a+1)}{(a-1)*(a+1)} + \frac{(a-1)*(a-1)}{(a+1)*(a-1)} &= \frac{a*a+a*1+1*a+1*1}{a*a+a*1-1*a-1*1} + \frac{a*a+a*(-1)-1*a-1*(-1)}{a*a+a*(-1)+1*a+1*(-1)} = \frac{a^2+2a+1}{a^2-1} + \frac{a^2-2a+1}{a^2-1} = \\
 &= \frac{(a^2+2a+1)+(a^2-2a+1)}{a^2-1} = \frac{a^2+2a+1+a^2-2a+1}{a^2-1} = \frac{2a^2+2}{a^2-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{a*(a+3)}{(a+4)*(a+3)} - \frac{a*(a+4)}{(a+3)*(a+4)} &= \frac{a^2+3a}{a^2+7a+12} - \frac{a^2+4a}{a^2+7a+12} = \frac{(a^2+3a)-(a^2+4a)}{a^2+7a+12} = \frac{-a}{a^2+7a+12}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } \frac{(3a-5)*(a-2)}{(a-1)*(a-2)} + \frac{(2a+3)*(a-1)}{(a-2)*(a-1)} &= \frac{3a*a+3a*(-2)-5*a-5*(-2)}{a*a+a*(-2)-1*a-1*(-2)} + \frac{2a*a+2a*(-1)+3*a+3*(-1)}{a*a+a*(-1)-2*a-2*(-1)} = \\
 &= \frac{3a^2-6a-5a+10}{a^2-2a-1a+2} + \frac{2a^2-2a+3a-3}{a^2-a-2a+2} = \frac{3a^2-11a+10}{a^2-3a+2} + \frac{2a^2+a-3}{a^2-3a+2} = \frac{3a^2-11a+10+2a^2+a-3}{a^2-3a+2} = \frac{5a^2-10a+7}{a^2-3a+2}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } \frac{(4-a)*(2-a)}{(4+a)*(2-a)} - \frac{(2+a)*(4+a)}{(2-a)*(4+a)} &= \frac{a^2-6a+8}{-a^2-2a+8} - \frac{a^2+6a+8}{-a^2-2a+8} = \frac{(a^2-6a+8)-(a^2+6a+8)}{-a^2-2a+8} = \frac{-12a}{-a^2-2a+8} = \\
 &= \frac{12a}{a^2+2a-8}
 \end{aligned}$$

6.5

$$\begin{aligned}
 \text{a. } \frac{a}{a-b} - \frac{b}{a-2b} &= \frac{a*(a-2b)}{(a-b)*(a-2b)} - \frac{b*(a-b)}{(a-2b)*(a-b)} = \frac{a^2-2ab}{a^2-2ab-ba+2b^2} - \frac{ab-b^2}{a^2-ab-2ab+2b^2} = \\
 &= \frac{a^2-2ab}{a^2-3ab+2b^2} - \frac{ab-b^2}{a^2-3ab+2b^2} = \frac{(a^2-2ab)-(ab-b^2)}{a^2-3ab+2b^2} = \frac{a^2-2ab-ab+b^2}{a^2-3ab+2b^2} = \frac{a^2+b^2-3ab}{a^2-3ab+2b^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{1}{a-b} + \frac{1}{a+b} &= \frac{a+b}{(a-b)(a+b)} + \frac{a-b}{(a+b)(a-b)} = \frac{a+b}{a^2-b^2} + \frac{a-b}{a^2-b^2} = \frac{(a+b)+(a-b)}{a^2-b^2} = \frac{a+b+a-b}{a^2-b^2} = \frac{2a}{a^2-b^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{2}{a-b} - \frac{2a}{a-2} &= \frac{2(a-2)}{(a-b)(a-2)} - \frac{(2a)(a-b)}{(a-2)(a-b)} = \frac{2a-4}{a^2-2a-ab+2b} - \frac{2a^2-2ab}{a^2-2a-ab+2b} = \frac{(2a-4)-(2a^2-2ab)}{a^2-2a-ab+2b} = \\
 &= \frac{2a-4-2a^2+2ab}{a^2-2a-ab+2b} = \frac{-2a^2+2ab+2a-4}{a^2-2a-ab+2b}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } \frac{1}{a-b} + \frac{a}{2a+3b} &= \frac{(2a+3b)}{(a-b)(2a+3b)} + \frac{a(a-b)}{(2a+3b)(a-b)} = \frac{2a+3b}{2a^2+3ab-2ab-3b^2} + \frac{a^2-ab}{2a^2+3ab-2ab-3b^2} = \\
 &= \frac{(2a+3b)+(a^2-ab)}{2a^2+ab-3b^2} = \frac{2a+3b+a^2-ab}{(2a^2+ab-3b^2)} = \frac{a^2+2a+3b-ab}{2a^2+ab-3b^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } \frac{a+b}{a-3} - \frac{a-b}{a+3} &= \frac{(a+b)(a+3)}{(a-3)(a+3)} - \frac{(a-b)(a-3)}{(a+3)(a-3)} = \frac{a^2+3a+ab+3b}{a^2-9} - \frac{a^2-3a-ab+3b}{a^2-9} = \\
 &= \frac{(a^2+3a+ab+3b)-(a^2-3a-ab+3b)}{a^2-9} = \frac{(a^2+3a+ab+3b-a^2+3a+ab-3b)}{a^2-9} = \frac{2ab+6a}{a^2-9}
 \end{aligned}$$

6.6

$$\begin{aligned}
 \text{a. } \frac{a+b}{a-c} - \frac{a-b}{a+c} &= \frac{(a+b)(a+c)}{(a-c)(a+c)} - \frac{(a-b)(a-c)}{(a+c)(a-c)} = \frac{(a^2+ac+ab+bc)}{a^2+ac-ac-c^2} - \frac{a^2-ab-ac+bc}{a^2+ac-ac-c^2} = \frac{a^2+ac+ab+bc}{a^2-c^2} - \\
 &= \frac{a^2-ab-ac+bc}{a^2-c^2} = \frac{(a^2+ac+ab+bc)-(a^2-ab-ac+bc)}{a^2-c^2} = \frac{a^2+ac+ab+bc-a^2+ab+ac-bc}{a^2-c^2} = \frac{2ab+2ac}{a^2-c^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{2a+1}{a-b} + \frac{a-2}{a+b} &= \frac{(2a+1)(a+b)}{(a-b)(a+b)} + \frac{(a-2)(a-b)}{(a+b)(a-b)} = \frac{2a^2+2ab+a+b}{a^2+ab-ab-b^2} + \frac{a^2-ab-2a+2b}{a^2+ab-ab-b^2} = \\
 &= \frac{(2a^2+2ab+a+b)+(a^2-ab-2a+2b)}{a^2-b^2} = \frac{2a^2+2ab+a+b+a^2-ab-2a+2b}{a^2-b^2} = \frac{3a^2+ab-a+3b}{a^2-b^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{4-a}{a+4b} - \frac{ab}{4a+b} &= \frac{(4-a)(4a+b)}{(a+4b)(4a+b)} - \frac{(ab)(a+4b)}{(a+4b)(a+4b)} = \frac{16a+4b-4a^2-ab}{4a^2+ab+16ab+4b^2} - \frac{a^2b+4ab^2}{4a^2+ab+16ab+4b^2} = \\
 &= \frac{-4a^2-ab+16a+4b}{4a^2+17ab+4b^2} - \frac{a^2b+4ab^2}{4a^2+17ab+4b^2} = \frac{(-4a^2-ab+16a+4b)-(a^2b+4ab^2)}{4a^2+17ab+4b^2} = \\
 &= \frac{-a^2b-4ab^2-4a^2-ab+16a+4b}{4a^2+17ab+4b^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } \frac{a-5c}{b-c} + \frac{2b+3}{a-b} &= \frac{(a-5c)(a-b)}{(b-c)(a-b)} + \frac{(2b+3)(b-c)}{(a-b)(a-c)} = \frac{a^2-ab-5ac+5bc}{ab-b^2-ac+bc} + \frac{2b^2-2bc+3b-3c}{ab-b^2-ac+bc} = \\
 &= \frac{(a^2-ab-5ac+5bc)+(2b^2-2bc+3b-3c)}{ab-b^2-ac+bc} = \frac{a^2+2b^2-ab-5ac+3bc+3b-3c}{-b^2+ab-ac+bc}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } \frac{a}{4+a+b} - \frac{2+a}{4-a+b} &= \frac{a(4-a+b)}{(4+a+b)(4-a+b)} - \frac{(2+a)(4+a+b)}{(4-a+b)(4+a+b)} = \frac{4a-a^2+ab}{16-4a+4b+4a-a^2+ab+4b-ab+b^2} - \\
 &= \frac{8+2a+2b+4a+a^2+ab}{16-4a+4b+4a-a^2+ab+4b-ab+b^2} = \frac{4a-a^2+ab}{-a^2+b^2+8b+16} - \frac{a^2+6a+2b+ab+8}{-a^2+b^2+8b+16} = \\
 &= \frac{(4a-a^2+ab)-(a^2+6a+2b+ab+8)}{-a^2+b^2+8b+16} = \frac{4a-a^2+ab-a^2-6a-2b-ab-8}{-a^2+b^2+8b+16} = \frac{-2a^2-2a-2b-8}{-a^2+b^2+8b+16} = \frac{2a^2+2a+2b+8}{a^2-b^2-8b-16}
 \end{aligned}$$

6.7

$$\text{a. } \frac{3a+18}{9b-6} \rightarrow \text{alle termen delen door } 3 = \frac{a+6}{3b-2} \quad b \neq \frac{2}{3}$$

$$\text{b. } \frac{a^2+a}{a+1} = \frac{a(a+1)}{a+1} = a * \frac{a+1}{a+1} = a * 1 = a \quad a \neq -1$$

$$\text{c. } \frac{4a-2}{2a^2-a} = \frac{2(2a-1)}{a(2a-1)} = \frac{2}{a} \quad a \neq 0, a \neq \frac{1}{2}$$

$$\text{d. } \frac{a+2b}{a^2-4b^2} = \frac{a+2b}{(a+2b)(a-2b)} = \frac{1}{a-2b} \quad a \neq 2b, a \neq -2b$$

$$\text{e. } \frac{ab+b^3}{b^2-3b} = \frac{b(a+b^2)}{b(b-3)} = \frac{a+b^2}{b-3} \quad b \neq 3, b \neq 0$$

6.8

$$a. \quad \frac{a^2b+ab^2}{3abc} = \frac{ab(a+b)}{3abc} \rightarrow \text{teller en noemer delen door } ab \rightarrow \frac{a+b}{3c}$$

$$b. \quad \frac{a^2-4a}{a+2a^2} = \frac{a(a-4)}{a(1+2a)} = \frac{a-4}{1+2a}$$

$$c. \quad \frac{4ab-3ab^2}{a^2-abc} = \frac{a(4b-3b^2)}{a(a-bc)} = \frac{4b-3b^2}{a-bc}$$

$$d. \quad \frac{a^2+2ab+b^2}{a^2-b^2} = \frac{(a+b)(a+b)}{(a+b)(a-b)} = \frac{a+b}{a-b}$$

$$e. \quad \frac{a^4-b^2}{a^2-b} = \frac{(a^2+b)(a^2-b)}{a^2-b} = a^2 + b$$

6.9

$$a. \quad \frac{1}{a-3} - \frac{1}{a^2-9} = \frac{1}{(a-3)} - \frac{1}{(a+3)(a-3)} = \frac{a+3}{(a+3)(a-3)} - \frac{1}{(a+3)(a-3)} = \frac{a+3-1}{(a+3)(a-3)} = \frac{a+2}{a^2-9}$$

$$b. \quad \frac{1}{a-3} - \frac{a}{a^2-9} = \frac{1}{(a-3)} - \frac{a}{(a+3)(a-3)} = \frac{a+3}{(a+3)(a-3)} - \frac{a}{(a+3)(a-3)} = \frac{a+3-a}{(a+3)(a-3)} = \frac{3}{a^2-9}$$

$$c. \quad \frac{a^2+1}{a-3} - \frac{a^2-1}{a+3} = \frac{(a^2+1)(a+3)}{(a-3)(a+3)} - \frac{(a^2-1)(a-3)}{(a+3)(a-3)} = \frac{a^3+3a^2+a+3}{a^2-9} - \frac{a^3-3a^2-a+3}{a^2-9} = \frac{(a^3+3a^2+a+3)-(a^3-3a^2-a+3)}{a^2-9} = \frac{6a^2+2a}{a^2-9}$$

$$d. \quad \frac{b}{a-b} + \frac{a}{b-a} = \frac{b}{a-b} + \frac{a}{-(a-b)} = \frac{b}{a-b} - \frac{a}{a-b} = \frac{b-a}{a-b} = \frac{-(a-b)}{a-b} = -1$$

$$e. \quad \frac{a^2-1}{a-1} - \frac{a^2+1}{a+1} = \frac{(a+1)(a-1)}{a-1} - \frac{a^2+1}{a+1} = a + 1 - \frac{a^2+1}{a+1} = \frac{(a+1)^2}{a+1} - \frac{a^2+1}{a+1} = \frac{(a+1)^2-(a^2+1)}{a+1} = \frac{a^2+2a+1-a^2-1}{a+1} = \frac{2a}{a+1}$$

6.10

$$a. \quad \frac{a+b}{a-2b} - \frac{a-2b}{a+b} = \frac{(a+b)(a+b)}{(a-2b)(a+b)} - \frac{(a-2b)(a-2b)}{(a+b)(a-2b)} = \frac{a^2+ab+ab+b^2}{a^2+ab-2ab-2b^2} - \frac{a^2-2ab-2ab+4b^2}{a^2-2ab+ab-2b^2} = \frac{a^2+2ab+b^2}{a^2-ab-2b^2} - \frac{a^2-4ab+4b^2}{a^2-ab-2b^2} = \frac{(a^2+2ab+b^2)-(a^2-4ab+4b^2)}{a^2-ab-2b^2} = \frac{a^2+2ab+b^2-a^2+4ab-4b^2}{a^2-ab-2b^2} = \frac{6ab-3b^2}{a^2-ab-2b^2}$$

$$b. \quad \frac{a^2+ab}{a^2-b^2} + a - 1 = \frac{a(a+b)}{(a+b)(a-b)} + a - 1 = \frac{a}{a-b} + a - 1 = \frac{a}{a-b} + \frac{a}{1} - \frac{1}{1} = \frac{a}{a-b} + \frac{a(a-b)}{a-b} - \frac{a-b}{a-b} = \frac{(a+(a(a-b))-(a-b))}{a-b} = \frac{a+a^2-ab-a+b}{a-b} = \frac{a^2-ab+b}{a-b}$$

$$c. \quad \frac{a}{a^2-4} - \frac{2}{4-a^2} = \frac{a}{(a+2)(a-2)} - \frac{2}{(2+a)(2-a)} = \frac{a}{(a+2)(a-2)} - \frac{2}{-(a+2)(a-2)} = \frac{a}{(a+2)(a-2)} + \frac{2}{(a+2)(a-2)} = \frac{a+2}{(a+2)(a-2)} = \frac{1}{a-2}$$

$$d. \quad \frac{3a-2b}{a-b} + \frac{2a+3b}{3a} = \frac{(3a)(3a-2b)}{(a-b)(3a)} + \frac{((2a+3b)(a-b))}{(3a)(a-b)} = \frac{9a^2-6ab}{3a^2-3ab} + \frac{2a^2-2ab+3ab-3b^2}{3a^2-3ab} = \frac{(9a^2-6ab)+(2a^2+ab-3b^2)}{3a^2-3ab} = \frac{9a^2-6ab+2a^2+ab-3b^2}{3a^2-3ab} = \frac{11a^2-5ab-3b^2}{3a^2-3ab}$$

e. $\frac{4-a}{a} - \frac{4+a}{2a} = \frac{2(4-a)}{2a} - \frac{4+a}{2a} = \frac{8-2a}{2a} - \frac{4+a}{2a} = \frac{8-2a-(4+a)}{2a} = \frac{8-2a-4-a}{2a} = \frac{4-3a}{2a}$