Uitwerkingen hoofdstuk 6

Wiskunde 1

6.1

a. $\frac{a+3}{a-3}\rightarrow term 1: a, term 2: 3\rightarrow \frac{a}{a-3}+\frac{3}{a-3}$

b. $\frac{2a+3b}{a-b}\rightarrow term 1: 2a, term 2: 3b\rightarrow \frac{2a}{a-b}+\frac{3b}{a-b}$

c. $\frac{a^{2}+3a+1}{a^{2}-3}\rightarrow term 1: a^{2}, term 2:3a, term 3:1\rightarrow \frac{a^{2}}{a^{2}-3}+\frac{3a}{a^{2}-3}+\frac{1}{a^{2}-3}$

d. $\frac{2a-b+3}{ab-3}\rightarrow term 1: 2a, term 2:-b, term 3:3\rightarrow \frac{2a}{ab-3}-\frac{b}{ab-3}+\frac{3}{ab-3}$

e. $\frac{2-5a}{b-a^{3}}\rightarrow term 1:2, term 2: -5a\rightarrow \frac{2}{b-a^{3}}-\frac{5a}{b-a^{3}}$

6.2

a. $\frac{a^{2}+b^{2}}{a^{2}-b^{2}}\rightarrow term 1:a^{2}, term 2:b^{2}\rightarrow \frac{a^{2}}{a^{2}-b^{2}}+\frac{b^{2}}{a^{2}-b^{2}}$

b. $\frac{ab+bc-ca}{a-2b}\rightarrow term 1:ab, term 2: bc, term 3: -ca\rightarrow \frac{ab}{a-2b}+\frac{bc}{a-2b}-\frac{ca}{a-2b}$

c. $\frac{b^{2}-1}{a^{2}-1}\rightarrow term 1: b^{2}, term 2: -1\rightarrow \frac{b^{2}}{a^{2}-1}-\frac{1}{a^{2}-1}$

d. $\frac{4abc+5}{c-ab}\rightarrow term 1:4abc, term 2:5\rightarrow \frac{4abc}{c-ab}+\frac{5}{c-ab}$

e. $\frac{(5ab^{2}-abc)}{\left(ab-c\right)}\rightarrow term 1:5ab^{2}, term 2: -abc\rightarrow \frac{5ab^{2}}{ab-c}-\frac{abc}{ab-c}$

6.3

a. $\frac{1}{a-3}-\frac{1}{a+3}=\frac{1\*\left(a+3\right)}{\left(a-3\right)\*\left(a+3\right)}-\frac{1\*\left(a-3\right)}{\left(a+3\right)\*\left(a-3\right)}=\frac{a+3}{a\*a+a\*3-3\*a-3\*3}-\frac{a-3}{a\*a+a\*\left(-3\right)+3\*a+3\*\left(-3\right)}=\frac{a+3}{a^{2}+3a-3a-9}-\frac{a-3}{a^{2}-3a+3a-9}=\frac{a+3}{a^{2}-9}-\frac{a-3}{a^{2}-9}=\frac{\left(\left(a+3\right)-\left(a-3\right)\right)}{a^{2}-9}=\frac{a+3-a+3}{a^{2}-9}=\frac{6}{a^{2}-9}$

b. $\frac{1}{a-3}+\frac{1}{a+3}=\frac{1\*\left(a+3\right)}{\left(a-3\right)\*\left(a+3\right)}+\frac{1\*\left(a-3\right)}{\left(a+3\right)\*\left(a-3\right)}=\frac{a+3}{a\*a+a\*3-3\*a-3\*3}+\frac{a-3}{a\*a+a\*\left(-3\right)+3\*a+3\*\left(-3\right)}=\frac{a+3}{a^{2}+3a-3a-9}+\frac{a-3}{a^{2}-3a+3a-9}=\frac{a+3}{a^{2}-9}+\frac{a-3}{a^{2}-9}=\frac{\left(\left(a+3\right)+\left(a-3\right)\right)}{a^{2}-9}=\frac{a+3+a-3}{a^{2}-9}=\frac{2a}{a^{2}-9}$

c. $\frac{2}{a-3}-\frac{1}{a+3}=\frac{2\*\left(a+3\right)}{\left(a-3\right)\*\left(a+3\right)}-\frac{1\*\left(a-3\right)}{\left(a+3\right)\*\left(a-3\right)}=\frac{2a+6}{a\*a+a\*3-3\*a-3\*3}-\frac{a-3}{a\*a+a\*\left(-3\right)+3\*a+3\*\left(-3\right)}=\frac{2a+6}{a^{2}+3a-3a-9}-\frac{a-3}{a^{2}-3a+3a-9}=\frac{2a+6}{a^{2}-9}-\frac{a-3}{a^{2}-9}=\frac{\left(\left(2a+6\right)-\left(a-3\right)\right)}{a^{2}-9}=\frac{2a+6-a+3}{a^{2}-9}=\frac{a+9}{a^{2}-9}$

d. $ \frac{1}{a-3}+\frac{a}{a+3}=\frac{1\*\left(a+3\right)}{\left(a-3\right)\*\left(a+3\right)}+\frac{a\*\left(a-3\right)}{\left(a+3\right)\*\left(a-3\right)}=\frac{a+3}{a\*a+a\*3-3\*a-3\*3}+\frac{a^{2}-3a}{a\*a+a\*\left(-3\right)+3\*a+3\*\left(-3\right)}=\frac{a+3}{a^{2}+3a-3a-9}+\frac{a^{2}-3a}{a^{2}-3a+3a-9}=\frac{a+3}{a^{2}-9}+\frac{a^{2}-3a}{a^{2}-9}=\frac{\left(\left(a+3\right)+\left(a^{2}-3a\right)\right)}{a^{2}-9}=\frac{a+3+a^{2}-3a}{a^{2}-9}=\frac{a^{2}-2a+3}{a^{2}-9}$

e. $\frac{a}{a-3}-\frac{a}{a+3}=\frac{a\*\left(a+3\right)}{\left(a-3\right)\*\left(a+3\right)}-\frac{a\*\left(a-3\right)}{\left(a+3\right)\*\left(a-3\right)}=\frac{a^{2}+3a}{a\*a+a\*3-3\*a-3\*3}-\frac{a^{2}-3a}{a\*a+a\*\left(-3\right)+3\*a+3\*\left(-3\right)}=\frac{a^{2}+3a}{a^{2}+3a-3a-9}-\frac{a^{2}-3a}{a^{2}-3a+3a-9}=\frac{a^{2}+3a}{a^{2}-9}-\frac{a^{2}-3a}{a^{2}-9}=\frac{\left(\left(a^{2}+3a\right)-\left(a^{2}-3a\right)\right)}{a^{2}-9}=\frac{a^{2}+3a-a^{2}+3a}{a^{2}-9}=\frac{6a}{a^{2}-9}$

6.4

a. $\frac{\left(a+1\right)\*\left(a+3\right)}{\left(a-2\right)\*\left(a+3\right)}-\frac{\left(a-1\right)\*\left(a-2\right)}{\left(a+3\right)\*\left(a-2\right)}=\frac{a\*a+a\*3+1\*a+1\*3}{a\*a+a\*3-2\*a-2\*3}-\frac{a\*a+a\*\left(-2\right)-1\*a-1\*\left(-2\right)}{a\*a+a\*\left(-2\right)+3\*a+3\*\left(-2\right)}=\frac{a^{2}+4a+3}{a^{2}+a-6}-\frac{a^{2}-3a+2}{a^{2}+a-6}=\frac{\left(a^{2}+4a+3\right)-\left(a^{2}-3a+2\right)}{a^{2}+a-6}=\frac{a^{2}+4a+3-a^{2}+3a-2}{a^{2}+a-6}=\frac{7a+1}{a^{2}+a-6}$

b. $\frac{\left(a+1\right)\*\left(a+1\right)}{\left(a-1\right)\*\left(a+1\right)}+\frac{\left(a-1\right)\*\left(a-1\right)}{\left(a+1\right)\*\left(a-1\right)}=\frac{a\*a+a\*1+1\*a+1\*1}{a\*a+a\*1-1\*a-1\*1}+\frac{a\*a+a\*\left(-1\right)-1\*a-1\*\left(-1\right)}{a\*a+a\*\left(-1\right)+1\*a+1\*\left(-1\right)}=\frac{a^{2}+2a+1}{a^{2}-1}+\frac{a^{2}-2a+1}{a^{2}-1}=\frac{\left(a^{2}+2a+1\right)+\left(a^{2}-2a+1\right)}{a^{2}-1}=\frac{a^{2}+2a+1+a^{2}-2a+1}{a^{2}-1}=\frac{2a^{2}+2}{a^{2}-1}$

c. $\frac{a\*\left(a+3\right)}{\left(a+4\right)\*\left(a+3\right)}-\frac{a\*\left(a+4\right)}{\left(a+3\right)\*\left(a+4\right)}=\frac{a^{2}+3a}{a^{2}+7a+12}-\frac{a^{2}+4a}{a^{2}+7a+12}=\frac{\left(a^{2}+3a\right)-\left(a^{2}+4a\right)}{a^{2}+7a+12}=\frac{-a}{a^{2}+7a+12}$

d. $\frac{\left(3a-5\right)\*\left(a-2\right)}{\left(a-1\right)\*\left(a-2\right)}+\frac{\left(2a+3\right)\*\left(a-1\right)}{\left(a-2\right)\*\left(a-1\right)}=\frac{3a\*a+3a\*\left(-2\right)-5\*a-5\*\left(-2\right)}{a\*a+a\*\left(-2\right)-1\*a-1\*\left(-2\right)}+\frac{2a\*a+2a\*\left(-1\right)+3\*a+3\*\left(-1\right)}{a\*a+a\*\left(-1\right)-2\*a-2\*\left(-1\right)}=\frac{3a^{2}-6a-5a+10}{a^{2}-2a-1a+2}+\frac{2a^{2}-2a+3a-3}{a^{2}-a-2a+2}=\frac{3a^{2}-11a+10}{a^{2}-3a+2}+\frac{2a^{2}+a-3}{a^{2}-3a+2}=\frac{3a^{2}-11a+10+2a^{2}+a-3}{a^{2}-3a+2}=\frac{5a^{2}-10a+7}{a^{2}-3a+2}$

e. $\frac{\left(4-a\right)\*\left(2-a\right)}{\left(4+a\right)\*\left(2-a\right)}-\frac{\left(2+a\right)\*\left(4+a\right)}{\left(2-a\right)\*\left(4+a\right)}=\frac{a^{2}-6a+8}{-a^{2}-2a+8}-\frac{a^{2}+6a+8}{-a^{2}-2a+8}=\frac{\left(a^{2}-6a+8\right)-\left(a^{2}+6a+8\right)}{-a^{2}-2a+8}=\frac{-12a}{-a^{2}-2a+8}=\frac{12a}{a^{2}+2a-8}$

6.5

a. $ \frac{a}{a-b}-\frac{b}{a-2b}=\frac{a\*\left(a-2b\right)}{\left(a-b\right)\*\left(a-2b\right)}-\frac{b\*\left(a-b\right)}{\left(a-2b\right)\*\left(a-b\right)}=\frac{a^{2}-2ab}{a^{2}-2ab-ba+2b^{2}}-\frac{ab-b^{2}}{a^{2}-ab-2ab+2b^{2}}=\frac{a^{2}-2ab}{a^{2}-3ab+2b^{2}}-\frac{ab-b^{2}}{a^{2}-3ab+2b^{2}}=\frac{\left(a^{2}-2ab\right)-\left(ab-b^{2}\right)}{a^{2}-3ab+2b^{2}}=\frac{a^{2}-2ab-ab+b^{2}}{a^{2}-3ab+2b^{2}}=\frac{a^{2}+b^{2}-3ab}{a^{2}-3ab+2b^{2}}$

b. $\frac{1}{a-b}+\frac{1}{a+b}=\frac{a+b}{\left(a-b\right)\left(a+b\right)}+\frac{a-b}{\left(a+b\right)\left(a-b\right)}=\frac{a+b}{a^{2}-b^{2}}+\frac{a-b}{a^{2}-b^{2}}=\frac{\left(a+b\right)+\left(a-b\right)}{a^{2}-b^{2}}=\frac{a+b+a-b}{a^{2}-b^{2}}=\frac{2a}{a^{2}-b^{2}}$

c. $\frac{2}{a-b}-\frac{2a}{a-2}=\frac{2\left(a-2\right)}{\left(a-b\right)\left(a-2\right)}-\frac{\left(2a\right)\left(a-b\right)}{\left(a-2\right)\left(a-b\right)}=\frac{2a-4}{a^{2}-2a-ab+2b}-\frac{2a^{2}-2ab}{a^{2}-2a-ab+2b}=\frac{\left(2a-4\right)-\left(2a^{2}-2ab\right)}{a^{2}-2a-ab+2b}=\frac{2a-4-2a^{2}+2ab}{a^{2}-2a-ab+2b}=\frac{-2a^{2}+2ab+2a-4}{a^{2}-2a-ab+2b}$

d. $\frac{1}{a-b}+\frac{a}{2a+3b}=\frac{\left(2a+3b\right)}{\left(a-b\right)\left(2a+3b\right)}+\frac{a\left(a-b\right)}{\left(2a+3b\right)\left(a-b\right)}=\frac{2a+3b}{2a^{2}+3ab-2ab-3b^{2}}+\frac{a^{2}-ab}{2a^{2}+3ab-2ab-3b^{2}}=\frac{\left(2a+3b\right)+\left(a^{2}-ab\right)}{2a^{2}+ab-3b^{2}}=\frac{2a+3b+a^{2}-ab}{\left(2a^{2}+ab-3b^{2}\right)}=\frac{a^{2}+2a+3b-ab}{2a^{2}+ab-3b^{2}}$

e. $\frac{a+b}{a-3}-\frac{a-b}{a+3}=\frac{\left(a+b\right)\left(a+3\right)}{\left(a-3\right)\left(a+3\right)}-\frac{\left(a-b\right)\left(a-3\right)}{\left(a+3\right)\left(a-3\right)}=\frac{a^{2}+3a+ab+3b}{a^{2}-9}-\frac{a^{2}-3a-ab+3b}{a^{2}-9}=\frac{\left(a^{2}+3a+ab+3b\right)-\left(a^{2}-3a-ab+3b\right)}{a^{2}-9}=\frac{\left(a^{2}+3a+ab+3b-a^{2}+3a+ab-3b\right)}{a^{2}-9}=\frac{2ab+6a}{a^{2}-9}$

6.6

a. $\frac{a+b}{a-c}-\frac{a-b}{a+c}=\frac{\left(a+b\right)\left(a+c\right)}{\left(a-c\right)\left(a+c\right)}-\frac{\left(a-b\right)\left(a-c\right)}{\left(a+c\right)\left(a-c\right)}=\frac{\left(a^{2}+ac+ab+bc\right)}{a^{2}+ac-ac-c^{2}}-\frac{a^{2}-ab-ac+bc}{a^{2}+ac-ac-c^{2}}=\frac{a^{2}+ac+ab+bc}{a^{2}-c^{2}}-\frac{a^{2}-ab-ac+bc}{a^{2}-c^{2}}=\frac{\left(a^{2}+ac+ab+bc\right)-\left(a^{2}-ab-ac+bc\right)}{a^{2}-c^{2}}=\frac{a^{2}+ac+ab+bc-a^{2}+ab+ac-bc}{a^{2}-c^{2}}=\frac{2ab+2ac}{a^{2}-c^{2}}$

b. $\frac{2a+1}{a-b}+\frac{a-2}{a+b}=\frac{\left(2a+1\right)\left(a+b\right)}{\left(a-b\right)\left(a+b\right)}+\frac{\left(a-2\right)\left(a-b\right)}{\left(a+b\right)\left(a-b\right)}=\frac{2a^{2}+2ab+a+b}{a^{2}+ab-ab-b^{2}}+\frac{a^{2}-ab-2a+2b}{a^{2}+ab-ab-b^{2}}=\frac{\left(2a^{2}+2ab+a+b\right)+\left(a^{2}-ab-2a+2b\right)}{a^{2}-b^{2}}=\frac{2a^{2}+2ab+a+b+a^{2}-ab-2a+2b}{a^{2}-b^{2}}=\frac{3a^{2}+ab-a+3b}{a^{2}-b^{2}}$

c. $\frac{4-a}{a+4b}-\frac{ab}{4a+b}=\frac{\left(4-a\right)\left(4a+b\right)}{\left(a+4b\right)\left(4a+b\right)}-\frac{\left(ab\right)\left(a+4b\right)}{\left(4a+b\right)\left(a+4b\right)}=\frac{16a+4b-4a^{2}-ab}{4a^{2}+ab+16ab+4b^{2}}-\frac{a^{2}b+4ab^{2}}{4a^{2}+ab+16ab+4b^{2}}=\frac{-4a^{2}-ab+16a+4b}{4a^{2}+17ab+4b^{2}}-\frac{a^{2}b+4ab^{2}}{4a^{2}+17ab+4b^{2}}=\frac{\left(-4a^{2}-ab+16a+4b\right)-\left(a^{2}b+4ab^{2}\right)}{4a^{2}+17ab+4b^{2}}=\frac{-a^{2}b-4ab^{2}-4a^{2}-ab+16a+4b}{4a^{2}+17ab+4b^{2}}$

d. $\frac{a-5c}{b-c}\mp \frac{2b+3}{a-b}=\frac{\left(a-5c\right)\left(a-b\right)}{\left(b-c\right)\left(a-b\right)}+\frac{\left(2b+3\right)\left(b-c\right)}{\left(a-b\right)\left(a-c\right)}=\frac{a^{2}-ab-5ac+5bc}{ab-b^{2}-ac+bc}+\frac{2b^{2}-2bc+3b-3c}{ab-b^{2}-ac+bc}=\frac{\left(a^{2}-ab-5ac+5bc\right)+\left(2b^{2}-2bc+3b-3c\right)}{ab-b^{2}-ac+bc}=\frac{a^{2}+2b^{2}-ab-5ac+3bc+3b-3c}{-b^{2}+ab-ac+bc}$

e. $\frac{a}{4+a+b}-\frac{2+a}{4-a+b}=\frac{a\left(4-a+b\right)}{\left(4+a+b\right)\left(4-a+b\right)}-\frac{\left(2+a\right)\left(4+a+b\right)}{\left(4-a+b\right)\left(4+a+b\right)}=\frac{4a-a^{2}+ab}{16-4a+4b+4a-a^{2}+ab+4b-ab+b^{2}}-\frac{8+2a+2b+4a+a^{2}+ab}{16-4a+4b+4a-a^{2}+ab+4b-ab+b^{2}}=\frac{4a-a^{2}+ab}{-a^{2}+b^{2}+8b+16}-\frac{a^{2}+6a+2b+ab+8}{-a^{2}+b^{2}+8b+16}=\frac{\left(4a-a^{2}+ab\right)-\left(a^{2}+6a+2b+ab+8\right)}{-a^{2}+b^{2}+8b+16}=\frac{4a-a^{2}+ab-a^{2}-6a-2b-ab-8}{-a^{2}+b^{2}+8b+16}=\frac{-2a^{2}-2a-2b-8}{-a^{2}+b^{2}+8b+16}=\frac{2a^{2}+2a+2b+8}{a^{2}-b^{2}-8b-16}$

6.7

a. $\frac{3a+18}{9b-6}\rightarrow alle termen delen door 3=\frac{a+6}{3b-2} b\ne \frac{2}{3}$

b. $\frac{a^{2}+a}{a+1}=\frac{a\left(a+1\right)}{a+1}=a\*\frac{a+1}{a+1}=a\*1=a a\ne -1$

c. $\frac{4a-2}{2a^{2}-a}=\frac{2\left(2a-1\right)}{a\left(2a-1\right)}=\frac{2}{a} a\ne 0$,$ a\ne \frac{1}{2}$

d. $\frac{a+2b}{a^{2}-4b^{2}}=\frac{a+2b}{\left(a+2b\right)\left(a-2b\right)}=\frac{1}{a-2b}$ $a\ne 2b$,$ a\ne -2b$

e. $\frac{ab+b^{3}}{b^{2}-3b}=\frac{b\left(a+b^{2}\right)}{b\left(b-3\right)}=\frac{a+b^{2}}{b-3} b\ne 3$, $b\ne 0$

6.8

a. $\frac{a^{2}b+ab^{2}}{3abc}=\frac{ab(a+b)}{3abc}\rightarrow teller en noemer delen door ab\rightarrow \frac{a+b}{3c}$

b. $\frac{a^{2}-4a}{a+2a^{2}}=\frac{a(a-4)}{a(1+2a)}=\frac{a-4}{1+2a}$

c. $\frac{4ab-3ab^{2}}{a^{2}-abc}=\frac{a(4b-3b^{2})}{a(a-bc)}=\frac{4b-3b^{2}}{a-bc}$

d. $\frac{a^{2}+2ab+b^{2}}{a^{2}-b^{2}}=\frac{\left(a+b\right)\left(a+b\right)}{\left(a+b\right)\left(a-b\right)}=\frac{a+b}{a-b}$

e. $\frac{a^{4}-b^{2}}{a^{2}-b}=\frac{\left(a^{2}+b\right)\left(a^{2}-b\right)}{a^{2}-b}=a^{2}+b$

6.9

a. $\frac{1}{a-3}-\frac{1}{a^{2}-9}=\frac{1}{\left(a-3\right)}-\frac{1}{(a+3)\left(a-3\right)}=\frac{a+3}{(a+3)\left(a-3\right)}-\frac{1}{(a+3)\left(a-3\right)}=\frac{a+3-1}{(a+3)\left(a-3\right)}=\frac{a+2}{a^{2}-9}$

b. $\frac{1}{a-3}-\frac{a}{a^{2}-9}=\frac{1}{\left(a-3\right)}-\frac{a}{\left(a+3\right)\left(a-3\right)}=\frac{a+3}{\left(a+3\right)\left(a-3\right)}-\frac{a}{\left(a+3\right)\left(a-3\right)}=\frac{a+3-a}{\left(a+3\right)\left(a-3\right)}=\frac{3}{a^{2}-9}$

c. $\frac{a^{2}+1}{a-3}-\frac{a^{2}-1}{a+3}=\frac{\left(a^{2}+1\right)\left(a+3\right)}{\left(a-3\right)\left(a+3\right)}-\frac{\left(a^{2}-1\right)\left(a-3\right)}{\left(a+3\right)\left(a-3\right)}=\frac{a^{3}+3a^{2}+a+3}{a^{2}-9}-\frac{a^{3}-3a^{2}-a+3}{a^{2}-9}=\frac{\left(a^{3}+3a^{2}+a+3\right)-\left(a^{3}-3a^{2}-a+3\right)}{a^{2}-9}=\frac{a^{3}+3a^{2}+a+3-a^{3}+3a^{2}+a-3}{a^{2}-9}=\frac{6a^{2}+2a}{a^{2}-9}$

d. $\frac{b}{a-b}+\frac{a}{b-a}=\frac{b}{a-b}+\frac{a}{-\left(a-b\right)}=\frac{b}{a-b}-\frac{a}{a-b}=\frac{b-a}{a-b}=\frac{-(a-b)}{a-b}=-1$

e. $\frac{a^{2}-1}{a-1}-\frac{a^{2}+1}{a+1}=\frac{\left(a+1\right)\left(a-1\right)}{a-1}-\frac{a^{2}+1}{a+1}=a+1-\frac{a^{2}+1}{a+1}=\frac{\left(a+1\right)^{2}}{a+1}-\frac{a^{2}+1}{a+1}=\frac{\left(a+1\right)^{2}-\left(a^{2}+1\right)}{a+1}$= $\frac{a^{2}+2a+1-a^{2}-1}{a+1}=\frac{2a}{a+1}$

6.10

a. $\frac{a+b}{a-2b}-\frac{a-2b}{a+b}=\frac{\left(a+b\right)\left(a+b\right)}{\left(a-2b\right)\left(a+b\right)}-\frac{\left(a-2b\right)\left(a-2b\right)}{\left(a+b\right)\left(a-2b\right)}=\frac{a^{2}+ab+ab+b^{2}}{a^{2}+ab-2ab-2b^{2}}-\frac{a^{2}-2ab-2ab+4b^{2}}{a^{2}-2ab+ab-2b^{2}}=\frac{a^{2}+2ab+b^{2}}{a^{2}-ab-2b^{2}}-\frac{a^{2}-4ab+4b^{2}}{a^{2}-ab-2b^{2}}=\frac{\left(a^{2}+2ab+b^{2}\right)-\left(a^{2}-4ab+4b^{2}\right)}{a^{2}-ab-2b^{2}}=\frac{a^{2}+2ab+b^{2}-a^{2}+4ab-4b^{2}}{a^{2}-ab-2b^{2}}=\frac{6ab-3b^{2}}{a^{2}-ab-2b^{2}}$

b. $\frac{a^{2}+ab}{a^{2}-b^{2}}+a-1=\frac{a\left(a+b\right)}{\left(a+b\right)\left(a-b\right)}+a-1=\frac{a}{a-b}+a-1=\frac{a}{a-b}+\frac{a}{1}-\frac{1}{1}=\frac{a}{a-b}+\frac{a\left(a-b\right)}{a-b}-\frac{a-b}{a-b}=\frac{\left(a+\left(a\left(a-b\right)\right)-\left(a-b\right)\right)}{a-b}=\frac{a+a^{2}-ab-a+b}{a-b}=\frac{a^{2}-ab+b}{a-b}$

c. $\frac{a}{a^{2}-4}-\frac{2}{4-a^{2}}=\frac{a}{\left(a+2\right)\left(a-2\right)}-\frac{2}{\left(2+a\right)\left(2-a\right)}=\frac{a}{\left(a+2\right)\left(a-2\right)}-\frac{2}{-\left(a+2\right)\left(a-2\right)}=\frac{a}{\left(a+2\right)\left(a-2\right)}+\frac{2}{\left(a+2\right)\left(a-2\right)}=\frac{a+2}{\left(a+2\right)\left(a-2\right)}=\frac{1}{a-2}$

d. $\frac{3a-2b}{a-b}+\frac{2a+3b}{3a}=\frac{\left(3a\right)\left(3a-2b\right)}{\left(a-b\right)\left(3a\right)}+\frac{\left(\left(2a+3b\right)\left(a-b\right)\right)}{\left(3a\right)\left(a-b\right)}=\frac{9a^{2}-6ab}{3a^{2}-3ab}+\frac{2a^{2}-2ab+3ab-3b^{2}}{3a^{2}-3ab}=\frac{\left(9a^{2}-6ab\right)+\left(2a^{2}+ab-3b^{2}\right)}{3a^{2}-3ab}=\frac{9a^{2}-6ab+2a^{2}+ab-3b^{2}}{3a^{2}-3ab}=\frac{11a^{2}-5ab-3b^{2}}{3a^{2}-3ab}$

e. $\frac{4-a}{a}-\frac{4+a}{2a}=\frac{2\left(4-a\right)}{2a}-\frac{4+a}{2a}=\frac{8-2a}{2a}-\frac{4+a}{2a}=\frac{8-2a-\left(4+a\right)}{2a}=\frac{8-2a-4-a}{2a}=\frac{4-3a}{2a}$